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## Cohesive Sediment Entrainment Rate Functions: Expanding and Quantifying Their Parameterizations

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14. ABSTRACT  The erosion characteristics of cohesive sediments in nearshore environments depend on a number of biogeochemical processes. It is standard practice to experimentally determine the entrainment rate of sediment into the water column as a function of the applied bottom shear stress and fit this data to a power law function. This work addresses the inherent problems of fitting entrainment rate data to parameterized functions, focusing on the non-uniqueness of the power law approach and its sensitivity to errors in the data. An approach for incorporating the effects of bioturbation and consolidation, thus expanding the parameter space of the entrainment function, into the standard power law formulation is examined with regard to its physical sensitivity and its expanded predictive capability of biogeochemical changes in cohesive sediments. Finally, power law and exponential entrainment functions are compared in order to illustrate the physical sensitivity of a particular entrainment parameterization.					
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## 1. Introduction

The transport of cohesive sediments in nearshore environments such as bays and estuaries presents significant challenges to the modeling community (Violeau et al. 2002). Predicting the movement of sediment and pollutants in estuaries and at the mouths of rivers under a variety of conditions is a long-term goal of sediment transport modeling. Because cohesive sediments can remain suspended for long durations, they can be transported great distances by currents and significantly affect the nearshore environment. In particular, they can reduce the water clarity and transport pollutants far from their point of origin (Mehta 1989a). In order to better manage coastal regions, it is necessary to understand the physical and biological processes (i.e. mineralogy, salinity, organic carbon content, bioturbation) that affect the entrainment of cohesive sediments and incorporate them into predictive models. The endeavor of creating realistic models for cohesive sediment transport requires a large-scale community effort to understand the physics of these sediments and the environments in which they are created. This work examines one approach to this problem and the inherent problems of increasing the complexity of entrainment functions typically applied to cohesive sediment transport. The goal of this work is to illustrate a general methodology (not an overall solution) and how it can lead to more robust cohesive sediment entrainment functions.

The sediments in coastal environments experience forces generated by wave, wind, and tidal action (Dronkers and Miltenburg 1996). Sediments are suspended by turbulence within the bottom boundary layer and are transported by the mean flow, eventually settling out of suspension. These physical processes are not well understood and are currently areas of intense research in the scientific and engineering communities.

Consequently, realistic coupled model systems to describe these environments are only now being developed (Bruens et al. 2002, Peterson and Vested 2002, Schweim et al. 2002). In addition, and of equal importance, biogeochemical processes, flocculation, and consolidation occur in the water column and within seafloor sediments. These processes determine the material properties of the sediments at the water-sediment interface and thus determine their entrainment properties (Mehta et al. 1989b). The influence of biological and physical processes on mudflats has been examined in the INTRMUD project (Black et al., 1998), which demonstrated the importance of biostabilization on entrainment (de Brouwer et al., 2000; Droppo et al. 2001). The LISP project (Littoral Investigation of Sediment Properties) (Daborn, 1991) also examined the complex interaction of microflora and fauna in the intertidal environment. The biostabilization effect of biofilms is opposed by the destabilization caused by bioturbation (Grant and Daborn, 1994; Green et al., 2002). Laboratory work with natural sediments has shown that cohesive sediment erodibility increases rapidly with the activity of infauna, presumably because of their destruction of inter-particle bonds and primary depositional fabric (Tsai and Lick, 1988; Lintern et al., 2002). The complex interaction of these biological processes with physical processes like hydrodynamics and consolidation necessitates the use of more realistic entrainment models and carefully planned measurement programs (Tolhurst et al., 2000; Lick et al., 1998).

It is therefore necessary to quantify the effects that the pre- and post-depositional environments have on the physical properties of the sediments and incorporate these characteristics into predictive models through an entrainment function. The goal of this work is to demonstrate that one common approach to parameterizing entrainment

functions is not sufficiently robust to capture the complex processes that determine how cohesive sediments erode in nearshore environments. The broad spectrum of biogeochemical processes that occur within marine sediments require that entrainment functions be enhanced with more degrees of freedom to account for the complex time-dependent entrainment characteristics of marine sediments. In lieu of first-principles models that can predict the microscopic properties of cohesive sediments an empirical fitting procedure is still necessary, but such an approach must account for a broad range of sediment characteristics and how they evolve after deposition. Once again we emphasize that this is an example of a new kind of methodology for approaching entrainment functions rather than an overall solution to this very complex problem.

Power law parameterizations (e.g. Kandiah 1974, Lavelle et al. 1984, Lick et al. 1995) of the entrainment rate will be investigated in order to demonstrate the inherent problems of using entrainment functions with small parameter spaces. In particular, the non-uniqueness of this formulation and the problems of fitting it to data will be discussed. An approach that incorporates the effects of bioturbation and consolidation into the entrainment rate function will be examined as well (Keen and Furukawa 2006, hereinafter KF06). The KF06 approach for parameterizing the entrainment rate function demonstrates that more physical models of entrainment rate parameters can be developed for marine sediments. Such an approach provides the entrainment function with a certain degree of sensitivity to the physical environment and post-depositional evolution of the sediments which can only be improved through better models of the sediment physiochemical properties. We will also address the issue of different entrainment



parameterizations as well as the development of more fundamental models for cohesive sediment entrainment.

## 2. Fundamental Entrainment Rate Formulation

Commonly used power law formulations for the entrainment or erosion rate ( $\text{kg m}^{-2} \text{s}^{-1}$ ) of cohesive sediments are typically expressed in terms of an erosion constant and the critical shear stress. For this work we refer to entrainment rate functions that have the general form:

$$E_B = A_0 \left( \frac{\tau}{\tau_c} - 1 \right)^m = A_0 (\tau^*)^m \quad (1)$$

where  $\tau_c$ ,  $A_0$ ,  $m$  are empirically determined constants (Lick et al. 1995);  $A_0$  has units of  $\text{kg m}^{-2} \text{s}^{-1}$ ;  $\tau$  and  $\tau_c$  (Pa) are, respectively, the bottom shear stress and the critical shear stress at which entrainment occurs. The excess shear stress, given by the term in parentheses, is represented by  $\tau^*$ . Equation (1) expresses the dependence of the sediments on the excess shear stress as well as the intrinsic entrainment properties of the material through the coefficient  $A_0$ . In principle,  $\tau_c$ ,  $A_0$ , and  $m$  should be functions of sediment properties such as water content, time since deposition, bioturbation, organic content, mineralogy, salinity, and floc size (see Partheniades 1986, for a review of the mechanical and chemical properties of bottom sediments and their effects on erosion). However, these dependencies are not explicitly treated in Equation (1). In some cases, the critical shear stress is given a dependency on depth below the seafloor because it is known to increase with consolidation and dewatering (Mehta et al., 1989b). A standard procedure for

determining the parameters  $\tau_c$ ,  $A_0$ , and  $m$  for a particular sediment sample is to fit Equation (1) to the entrainment rate data as a function of the applied shear stress (Lick et al. 1995). Because the dimensionless power  $m$  is determined by fitting Equation (1) to a particular data set its relationship to the sediment properties (and thus  $\tau_c$  and  $A_0$ ) is poorly understood and lacks a physically intuitive basis for its value (Lick et al., 1995). Unfortunately, as will be discussed further, the fitting procedure ignores the physical relationships that must certainly exist between these parameters and essentially lumps much of the sediment's physical complexity into the erosion constant and critical shear stress parameters. It is the conclusion of this study that such an approach is simply too limited in its formulation for realistic sediments that are subjected to a variety of time-dependent physical and biogeochemical processes.

To illustrate the essential problem of fitting field measurements to power laws we examine the non-uniqueness of Equation (1), which can be rewritten as

$$\tau^* = e^{\frac{-\ln(\frac{A_0}{E_B})}{m}} = \left( \frac{A_0}{E_B} \right)^{-\frac{1}{m}} \quad (2)$$

Using this formulation, a surface of constant entrainment rate (Figure 1) can be plotted to demonstrate that there are an infinite number of possibilities for the parameters  $\tau^*$ ,  $A_0$ , and  $m$  (assuming that  $\tau^*$  and  $A_0$  are always positive) for a specified value of the entrainment rate ( $E_B = 0.1 \text{ kg m}^{-2} \text{ s}^{-1}$ ). The ranges of the  $x$  and  $y$  axes were chosen to coincide with values of  $A_0$  and  $m$  that are reasonable for fitting known data sets (KF06). From the analysis of several data sets in KF06 it is clear that  $A_0$  can vary over several orders of magnitude and that  $m$  will be on the order of  $\sim 1$  to  $3$ . For  $E_B$  equal to  $0.1 \text{ kg m}^{-2}$

$\text{s}^{-1}$ ,  $\tau^*$  is observed to decrease with increasing values of both  $A_0$  and  $m$ . Figure 1 demonstrates that it is possible to have sediments with very different physical properties eroding at the same rate, even if they are from the same sample suite, and indicates that Equation (1) should be used with caution when analyzing entrainment data. If it is assumed that both  $\tau$  and  $\tau_c$  vary spatially and temporally during an experiment (Parchure and Mehta 1985), then it is conceivable that  $A_0$  and  $m$  are also changing in space and time. Therefore if  $\tau^*$ ,  $A_0$ , and  $m$  are assumed to be dependent not only on the sediment properties but also on each other, a much more complex and robust entrainment rate function can be derived.

Figure 1 indicates that  $\tau^*$  decreases with increasing  $A_0$  and  $m$ , although this is only for a small range of the potential values for  $A_0$  and  $m$ . Figure 2 is a two-dimensional projection of the isosurface plotted in Figure 1, but for a different range of  $A_0$ . The same value of the entrainment rate ( $E_B = 0.1 \text{ kg m}^{-2} \text{ s}^{-1}$ ) is used for Figures 1 and 2. For  $A_0$  greater than  $E_B$ ,  $\tau^*$  increases with increasing  $m$  for a constant value of  $A_0$ , whereas  $\tau^*$  decreases with increasing  $m$  for constant  $A_0$  less than  $E_B$ . Specifically, the contours of  $\tau^*$  change direction when the natural log of  $A_0/E_B$  changes sign, as indicated by Equation (2). This transition can be observed as  $A_0$  becomes greater than  $0.1 \text{ kg m}^{-2} \text{ s}^{-1}$  and the contour lines of  $\tau^*$  change from having negative slopes in the direction of increasing  $m$  to having positive slopes. The transition will always occur along the contour  $\tau^* = 1.0$  because the natural log of  $(A_0/E_B)$  is zero at the transition. This behavior is not unexpected from a mathematical point of view since  $A_0$  and  $m$  must adjust along a  $\tau^*$  isocontour if  $E_B$  is being held constant, but this sensitivity has not been addressed in the literature. Therefore, when experimental data are used to estimate the values of the

parameters in Equation (1), the range of  $A_0$  and  $m$  should be considered because it may have a significant impact on the results. In particular, uncertainties in observations of entrainment rate and excess shear stress can lead to different results for  $A_0$  and  $m$ , depending on how these quantities are related in the particular region of parameter space encompassed by the experiments. Additional constituent equations relating these parameters to the physical properties of the sediments as well as each other would make the solution procedure more complex but would nevertheless provide more physical constraints on how each parameter should vary.

Another way of viewing the sensitivity of the entrainment rate to  $\tau^*$ ,  $A_0$ , and  $m$  is to examine its behavior when  $\tau^*$  is held constant. Assuming a constant  $\tau^*$  equal to 9.0,  $A_0$  and  $m$  can have a range of values along an isocontour of the entrainment rate  $E_B$  (Figure 3). The contours of constant entrainment rate are plotted in Figure 4, which is a two dimensional projection of Figure 3. It can be seen from Figures 3 and 4 that  $E_B$  increases with increasing  $A_0$  and  $m$  as expected for  $\tau^*$  greater than 1. However, when  $\tau^*$  is less than 1, the contours have a very different structure (Figure 5). The isolines of  $E_B$  in Figure 4 are essentially orthogonal to those of Figure 5, indicating a significant change in the parameter space for  $\tau^*$  less than 1. These examples demonstrate that sediment samples with uniform excess shear stress and constant entrainment rates do not necessarily have to have uniform physical properties. Overall Figures 1-5 illustrate how insensitive Equation (1) is to the actual physical properties of the sediments and motivate a parameterization strategy that incorporates additional physics into the entrainment function.

Before discussing extensions to Equation (1), it is useful to examine representative surfaces of constant  $E_B$  in  $(\tau^*, A_0, m)$  parameter space (Figure 6). If Equation (1) is fit to a set of experimental data points, the resulting constant values of  $A_0$  and  $m$  represent a vertical line parallel to the  $\tau^*$  axis. This line intersects surfaces of constant entrainment rate measured for different values of excess shear stress. This is demonstrated in Figure 6 using isosurfaces of  $E_B$  equal to  $0.1 \text{ kg m}^{-2} \text{ s}^{-1}$  (lower surface) and  $0.5 \text{ kg m}^{-2} \text{ s}^{-1}$  (upper surface). Planes of constant  $\tau^*$  intersect these  $E_B$  isosurfaces along curved lines. Thus there are an infinite number of values for  $A_0$  and  $m$ , along each curve of constant  $\tau^*$ , for each  $E_B$  isosurface (as seen previously in Figure 2). Furthermore, measurement uncertainties in the entrainment rate and excess shear stress can produce very different values of  $A_0$  and  $m$ , depending on the geometry of the  $E_B$  isosurface. Measurement errors in  $\tau^*$  are more important in determining  $A_0$  and  $m$  if the  $E_B$  isosurface is slowly changing in parameter space. This is because small variations in  $\tau^*$  represent large changes in the values of  $A_0$  and  $m$  if the  $E_B$  isosurface varies slowly over a large area of the  $A_0$ - $m$  plane. As seen from Figure 6, different  $E_B$  isosurfaces can have very different gradients in parameter space with respect to how  $\tau^*$  changes with  $A_0$  and  $m$ . A more realistic model would allow  $A_0$  and  $m$  to vary along the  $\tau^*$  contours of each  $E_B$  isosurface while still satisfying Equation (1) for each  $(E_B, \tau^*)$  data point. This would be an important step in going beyond the standard curve fitting procedures that are used in the literature. In addition, such an approach would increase the correlation of  $A_0$  and  $m$  with the physical properties of the sediments.

### 3. Entrainment Rate Formulation with Bioturbation and Consolidation

Natural sediments experience a complex physical and biogeochemical environment, which affects their physical properties and thus the manner in which they erode. The previous section demonstrated that a simple parameterization of cohesive sediment entrainment rate such as Equation (1) does not have the capacity to describe the spatial and temporal changes that occur in the erosion characteristics of natural sediments in a robust manner. An initial approach for extending the entrainment rate function (KF06) incorporates physical and biological effects into the entrainment coefficient  $A_0$  of Equation (1). The  $A_0$  parameter can be expanded into three coefficients  $A_0 A_B A_C$ , in which  $A_0$  represents an entrainment parameter fundamental to a particular sediment type (i.e. the mineralogy),  $A_B$  represents the effects of bioturbation, and  $A_C$  represents the effects of consolidation.

In the KF06 study the consolidation factor was fit to an exponential function,

$$A_C = C_1 e^{(W - W_0)/t_1} \quad (3)$$

where:  $C_1 = 3.97 \times 10^{-3}$ ,  $t_1 = 1.99315$ ,  $W$  is the water content (%) of the sediment, and  $W_0 = 62.85$ . The values of  $C_1$ ,  $t_1$ , and  $W_0$  were derived by fitting Equation (3) to consolidated sediments from Lake Erie with no bioturbation (Fukuda and Lick 1980). Equation (3) is only valid for  $W$  ranging from 61.5 to 74% since this was the range of the water content data used to determine the parameters. It is apparent from Equation (3) that a decrease in  $W$  corresponds to increased consolidation and a lower entrainment rate. This parameterization is therefore expressing the physical effects of consolidation on the entrainment rate. The bioturbation factor is fit to a parabolic function of time,

$$A_B = C_2 + C_3 t + C_4 t^2 \quad (4)$$

where:  $C_2 = 5.093 \times 10^{-3}$  and  $C_3 = 2.186 \times 10^{-2} \text{ (day}^{-1}\text{)}$  and  $C_4 = 6.85 \times 10^{-4} \text{ (day}^{-2}\text{)}$ . The values of  $C_2$ ,  $C_3$ , and  $C_4$  are found by fitting Equation (4) to entrainment data for bioturbated sediments from the Tamar Estuary in the United Kingdom (Lintern et al. 2002). The interval of time over which bioturbation occurs is  $t$ , often interpreted as the time since deposition, and it is apparent from Equation (4) that the entrainment rate should increase with  $t$ . Equation (4) therefore incorporates the physical changes in the sediments (i.e. weakening of physical integrity) due to bioturbation. The methods and data used to derive the parameters  $A_B$  and  $A_C$  are discussed in detail in KF06. Note that the  $A_B$  and  $A_C$  parameters are dimensionless quantities. The expanded form of the entrainment rate function is

$$E_B = A_0 A_B A_C \left( \frac{\tau}{\tau_c} - 1 \right)^m \quad (5)$$

At this point we can focus on the sensitivity of the entrainment rate function to the parameters  $A_B$  and  $A_C$  as well as time since deposition  $t$  and water content  $W$ . It is the functional dependence of  $A_B$  and  $A_C$  on measurable physical quantities that leads to a more robust and realistic model of the entrainment rate. Although we are still using functions that have been fitted to a limited set of data they can describe a broader range of sediment entrainment properties. A much broader data base of sediment entrainment

measurements for a variety of different types of sediments is required for more generality and perhaps different functions for different ranges of water content and deposition time are required as well. The discussion in section (1) regarding the general properties of the parameter space for Equation (1) applies equally to Equation (5) and will not be repeated here.

The entrainment rate computed from Equation (5) is plotted as a function of both time and water content in Figures 7 and 8, respectively, in order to show the relative effects (i.e. magnitude) of bioturbation and consolidation. The ranges of  $t$  and  $W$  are similar to those used in KF06 to obtain the original parameterizations of  $A_B$  and  $A_C$ . The values of  $A_0$  and  $\tau^*$  are set equal to 1 and  $m = 0$  so that the curves in Figures 7 and 8 are equal to the values of  $A_B$  and  $A_C$  respectively. The entrainment rate is much more sensitive to small changes in water content due to the exponential form of  $A_C$  than it is to deposition time (and hence bioturbation). It takes approximately 25 days for  $E_B$  to equal  $1 \text{ kg m}^{-2} \text{ s}^{-1}$  in Figure 7 whereas a change of 4% in the water content (Figure 8) results in an equivalent change in the value of  $E_B$ . A contour plot of entrainment rate  $E_B$  as a function of time since deposition and water content (Figure 9), with  $A_0$  and  $\tau^*$  equal to 1.0, can be interpreted as representing the product  $A_B A_C$ . The conclusion to be drawn from Figure 9 is that sediments should be characterized with regard to their physical properties and post-depositional histories and not just their entrainment at different shear stress values. It is evident from Figure 9 that the same entrainment rate can be obtained for very different consolidation and bioturbation values and thus a prediction of the entrainment rate will depend on understanding the sediment properties and the environments in which they form. Increasing the sensitivity of entrainment functions to the material properties



of sediments therefore requires a deeper understanding of how sediments form and change in nearshore environments.

Perhaps the most direct way of comparing the relative effects of consolidation and bioturbation is to determine the entrainment rate as a function of excess shear stress with either bioturbation or consolidation affecting the sediment's erosion properties (Lavelle et al. 1984). Figure 10 plots the entrainment rate calculated from Equation (5) against excess shear stress with either bioturbation or consolidation activated. The deposition time of 5 days and water content of 70% are chosen to produce similar curves over a range of shear stress values with  $A_0 = 0.001 \text{ kg m}^{-2} \text{ s}^{-1}$  and  $m = 2.5$ . This leads again to the problem suggested by figure 9 of how to predict the entrainment rate if bioturbation and consolidation are both affecting the sediment properties. Both processes are present in natural sediments but it is often unclear what their individual contributions are to determining the properties of the bed.

Although the entrainment rate determined from Equation (5) depends on the post-depositional history through  $A_B$  and  $A_C$  there is still much more that could be done with respect to how *their* parameters evolve with time. This is a much larger issue which must eventually be addressed if robust predictive models of cohesive sediment transport are to be developed for nearshore environments. Although an approach similar to that employed in this paper for  $A_B$  and  $A_C$  could be used to create additional functional parameterizations, assuming that the data exists to construct such functions. This problem can be dealt with to some extent by adjusting  $A_0$ , but this requires an *ad hoc* (and unphysical) change in a fundamental parameter. The implicit problem with natural sediments is that both consolidation and bioturbation processes are time-dependent. In

addition, the final state of a sediment sample is dependent not only on its depositional history but also on its initial state with respect to infauna and water content. This discrepancy can only be addressed by more realistic models as well as more extensive field studies of natural marine sediments.

#### 4. Comparison of Power Law and Exponential Models of Entrainment Rate

The parameterizations included in Equation (1) and its extended version, Equation (5), have been examined in order to show their potential as well as their limitations. The limitations of Equation (1) with respect to the depth dependence of flocculated sediment have been discussed by Parchure and Mehta (1985). They found it difficult to fit their entrainment data to a power law such as Equation (1) and instead used an exponential function of shear stress with a depth-dependent critical shear stress  $\tau_c$ . This section compares Equation (1) to an exponential entrainment rate function in order to address the need for physical correlations between the various parameters of any proposed entrainment function.

The exponential entrainment rate function employed for this discussion has the following form:

$$E_B = \varepsilon_0 e^{\tau/\alpha} \quad (6)$$

where:  $\varepsilon_0$  ( $\text{kg m}^{-2} \text{s}^{-1}$ ) is the fundamental entrainment coefficient;  $\tau$  is the bottom shear stress; and  $\alpha$  is a parameter with units of shear stress. The entrainment rates calculated from Equations (1) and (6) are compared in Figure 11, which plots both as functions of

the shear stress  $\tau$ . The curves are obtained by setting  $A_0$  and  $\varepsilon_0$  equal to  $1 \text{ kg m}^{-2} \text{ s}^{-1}$  and adjusting the other parameters as needed. For the power law function, the  $m$  exponent is 2.4 and  $\tau_c$  is 0.05 Pa whereas the value of  $\alpha$  is 0.1 Pa for the exponential function. It should be noted that the values of entrainment rate in Figure 11 are not representative of actual field results and that both functions would have to be scaled down by lowering the values of  $A_0$  and  $\varepsilon_0$ . It should also be pointed out that Equation (6) only applies to  $\tau$  greater than  $\tau_c$ . Equation (6) never goes to zero unlike Equation (1) which is equal to zero when  $\tau$  equals  $\tau_c$ . The two curves have approximately the same magnitude over a small range of  $\tau$ , with the exponential entrainment exceeding the power law entrainment when  $\tau$  becomes greater than 0.55 Pa.

The fact that the two curves are similar over a narrow range of values is significant with regard to modeling entrainment. What physical significance do these parameters actually have if it is possible to make Equations (1) and (6) agree over some range of shear stress by simply adjusting their values? These adjustments must be arbitrarily determined because the parameters of each entrainment function are not correlated with each other in any physical way. This demonstrates the lack of any physical basis for the parameterizations of Equations (1) and (6) and further supports the need for empirically derived constitutive equations that govern the relationships between the parameters of an entrainment function.

## 5. Discussion

The increased parameterization of  $A_0$  presented in section 3 is intended to introduce more physical sensitivity to this parameter, although Equations (3) and (4) are still

limited by their lack of dynamical feedback with the local sedimentary environment. Because the water content and bioturbation parameters were determined using entrainment data dominated by those individual processes and not through dynamic models, it is impossible to gauge whether  $A_B$  or  $A_C$  is the dominant factor in the erosion of real cohesive sediments when both processes occur simultaneously. This would require the development of a coupled water column–seafloor benthic boundary layer model that could predict deposition, erosion, bioturbation, and consolidation, as well as their cumulative effects on sediment water content and structural properties (Kranenburg and Winterwerp 1997, Petersen and Vested 2002). A discussion of such models is beyond the scope of this paper, although they are the next logical step in cohesive sediment transport modeling.

As demonstrated by Figure 11 fitting parameterized functions to data is a non-trivial matter since the physical interpretation of their parameters can be uncertain. It is necessary to determine the physical or biological processes of most interest *a priori* and select a model that is based on them or neglect physical interpretations entirely. Furthermore, increasing the number of non-physical parameters (i.e. those with no direct physical interpretation) used in the entrainment function decreases its overall sensitivity to physical changes in the sediments. The  $m$  exponent in Equation (1) is a good example of a non-physical parameter. It is a non-dimensional number that will change based on some physical properties of the sediments, yet a physical interpretation of  $m$  with respect to sediment properties has not been explicitly discussed in the literature to the authors' knowledge.

## 6. Conclusion

The issues addressed in this work demonstrate the need for physics based approaches to cohesive sediment transport. As demonstrated in this study, power law entrainment rate functions do not have the ability to represent the broad range of erosion characteristics found in nearshore cohesive sediments. In order to model the erosion, transport, and deposition of cohesive sediments in a robust and realistic manner, the physical and biogeochemical processes that determine their physical properties must be understood at a more fundamental level.

The non-uniqueness of a power law entrainment rate function has been examined in order to demonstrate the inherent problems with fitting such functions to entrainment rate data. The results of this analysis clearly show that an isosurface of the entrainment rate could have an infinite number of possible values for  $A_0$  and  $m$  and still represent a single data point of  $(E_B, \tau^*)$ . This suggests that sediment properties can vary significantly in time and space and that fitting entrainment rate functions to only certain types of data may simply be ignoring the complex erosion characteristics of the sediment. In addition it also suggests that the parameters determined from one set of data may not be applicable to another set of data (taken at a different time or location). Furthermore, an analysis of multiple surfaces of entrainment rate indicates how restrictive the parameter fitting is with regard to the range of possible parameter values. These results demonstrate the need for entrainment rate functions that incorporate effects such as bioturbation and consolidation.

One approach for extending entrainment rate functions involves using measurable sediment properties such as water content and bioturbation to parameterize the

fundamental entrainment coefficient  $A_0$ . As discussed in this work, the entrainment coefficient  $A_0$  can be expanded into sub-coefficients (i.e.,  $A_0 A_B A_C$ ) that are fitted to data that represent the effects of either bioturbation or consolidation on the sediment sample. Although such an approach is limited by the data sets that are used as well as the inability of the fitted functions to respond dynamically to changing environmental conditions it is a step forward in making entrainment functions more physically realistic. The overall conclusion is that more fundamental and comprehensive coupled models of the water column-sea floor benthic boundary layer should be developed to predict sediment erosion and transport in complex nearshore environments.

Finally, a comparison of power law and exponential entrainment rate functions was performed in order to demonstrate the necessity for physics based parameterization schemes. It was found that different entrainment rate functions can have similar magnitudes over the same range of shear stress values if the parameters of each function are adjusted accordingly. This analysis demonstrated that the physical characteristics of the sediments are not well represented by arbitrary parameterizations (the general theme of this work) and that additional constitutive equations (derived from first principle models or empirical field data) are necessary to create physics based entrainment functions.

## **7. Acknowledgements**

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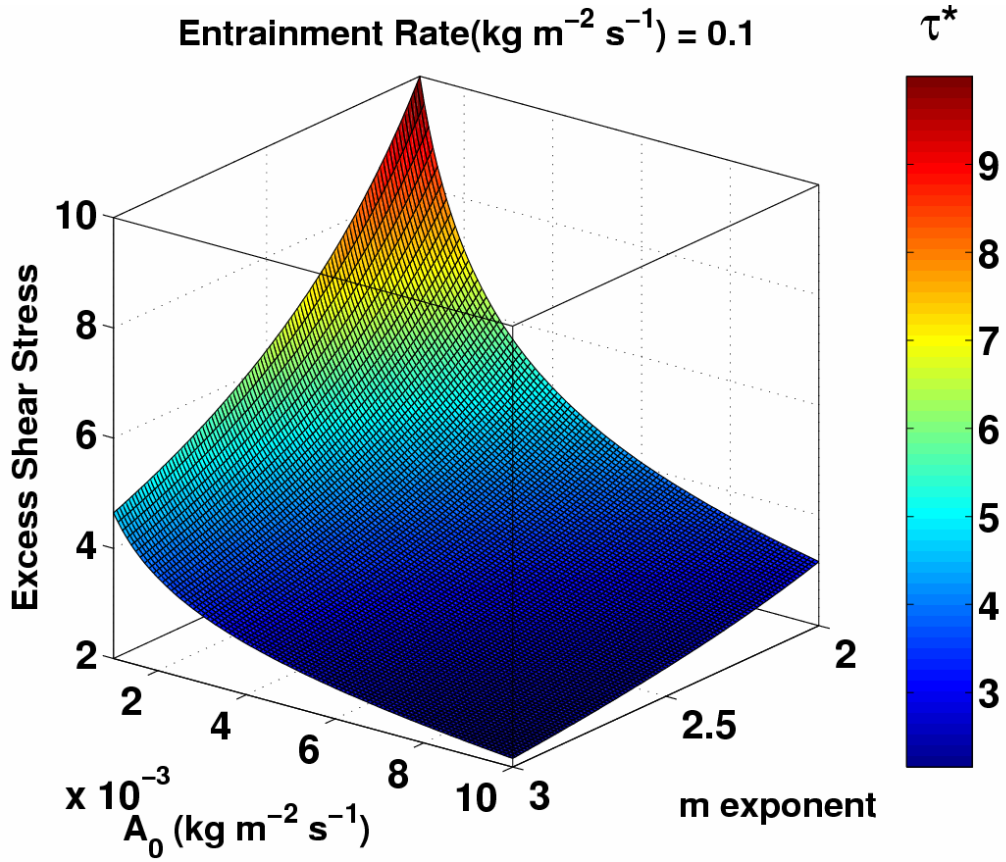


Figure 1. Isosurface of constant entrainment rate ( $E_B = 0.1 \text{ kg m}^{-2} \text{s}^{-1}$  for a range of  $\tau^*$ ,  $A_0$ , and  $m$ ).

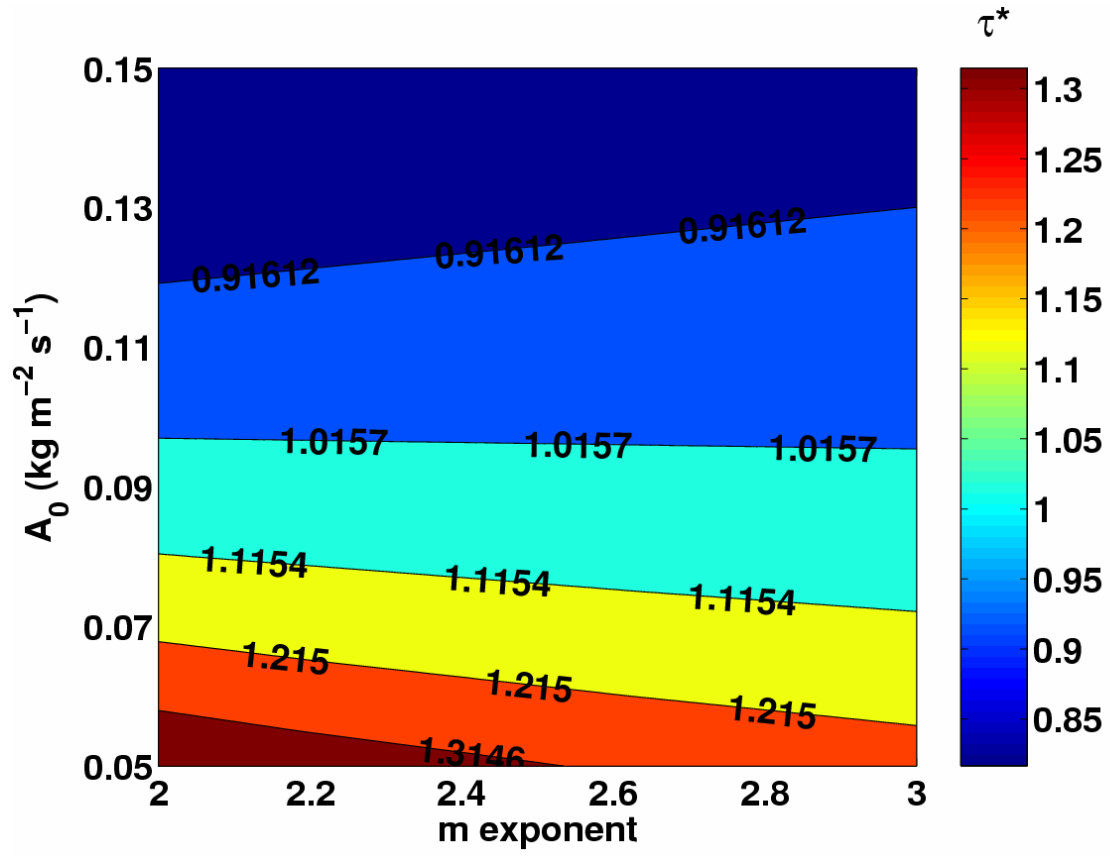


Figure.2. Contour plot of  $\tau^*$  as a function of  $A_0$  and  $m$  for  $E_B = 0.1 \text{ kg m}^{-2} \text{ s}^{-1}$ .

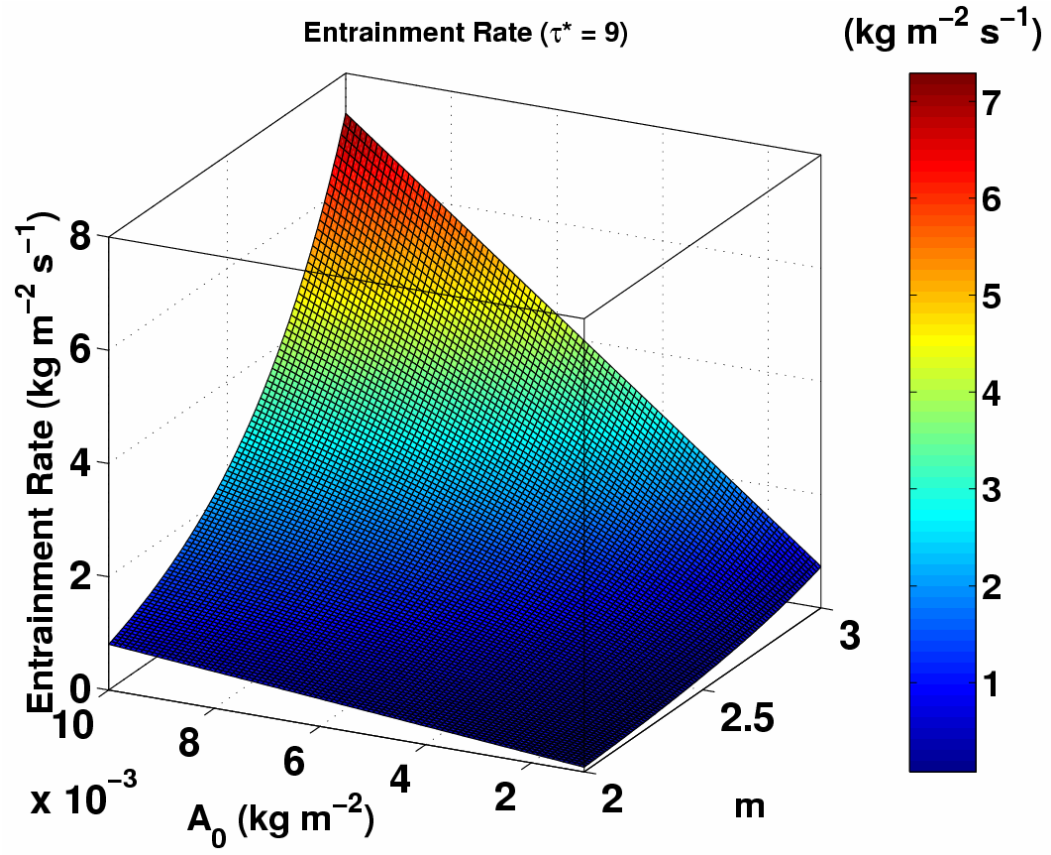


Figure 3.  $E_B$  (kg m<sup>-2</sup> s<sup>-1</sup>) as a function of  $A_0$  and  $m$  for a constant value of  $\tau^* = 9.0$ .

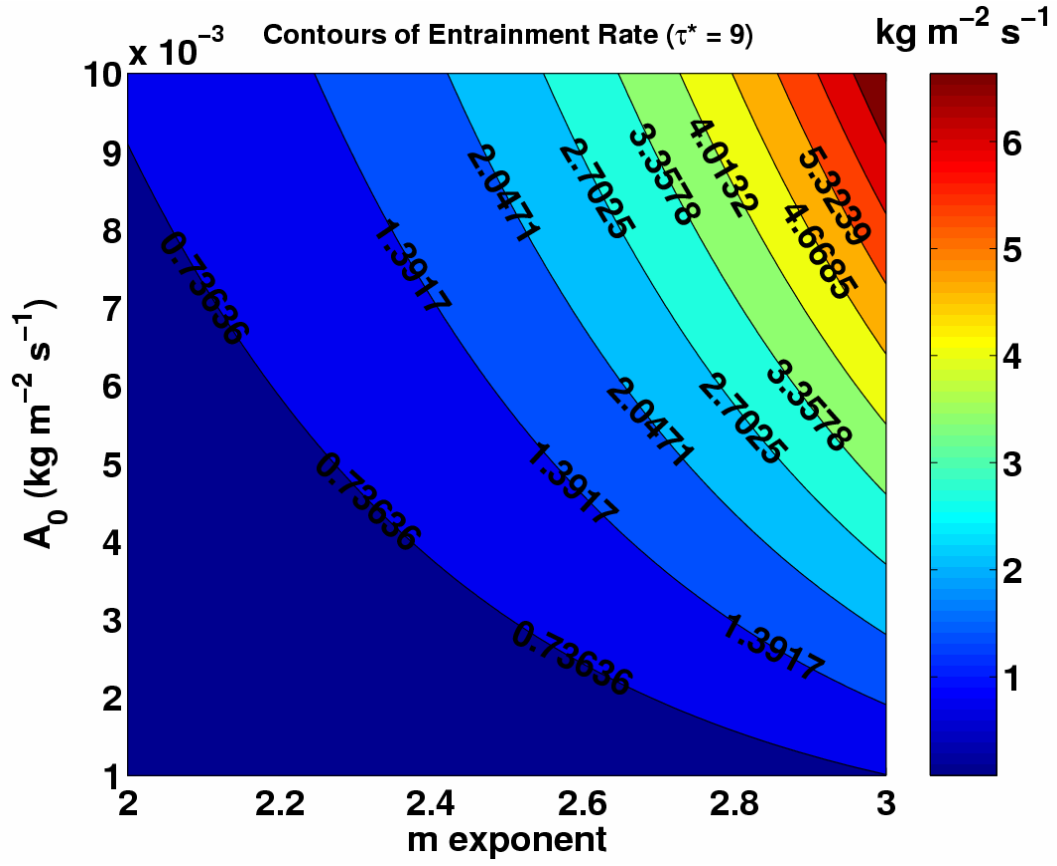


Figure 4. Contour plot of  $E_B$  ( $\text{kg m}^{-2} \text{s}^{-1}$ ) as a function of  $A_0$  and  $m$  for  $\tau^* = 9.0$ .

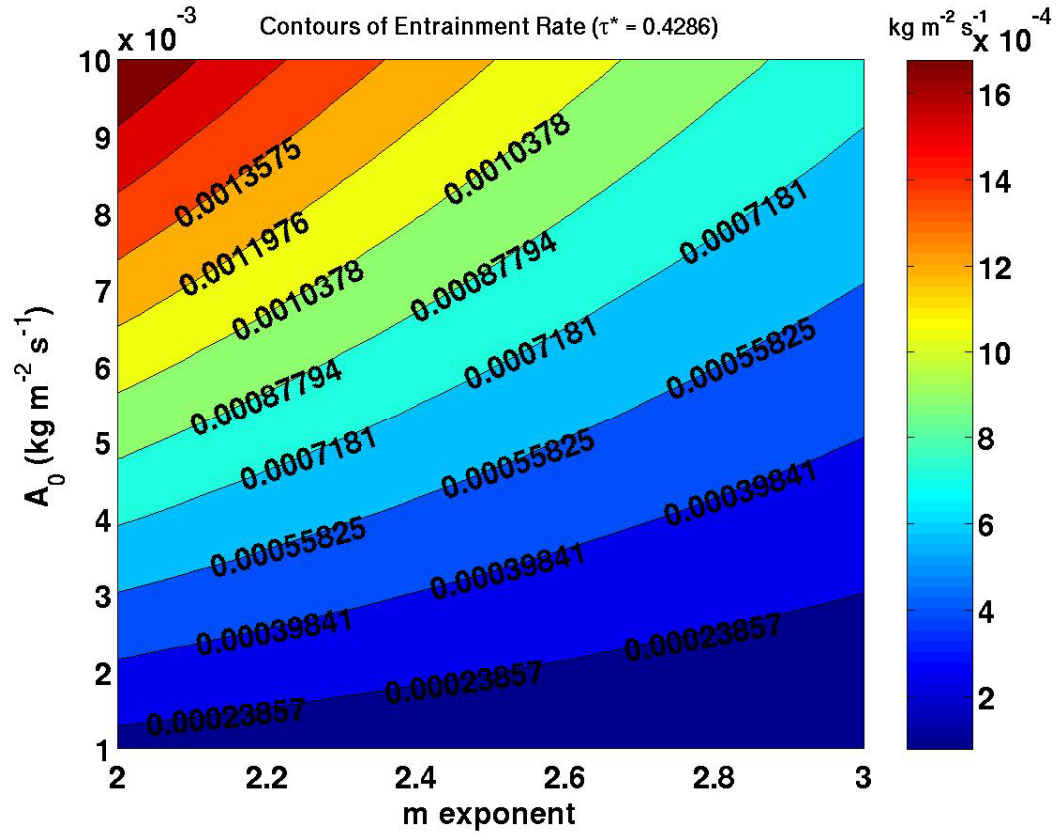


Figure 5. Contour plot of  $E_B$  ( $\text{kg m}^{-2} \text{s}^{-1}$ ) as a function of  $A_0$  and  $m$  for  $\tau^* = 0.4286$ .

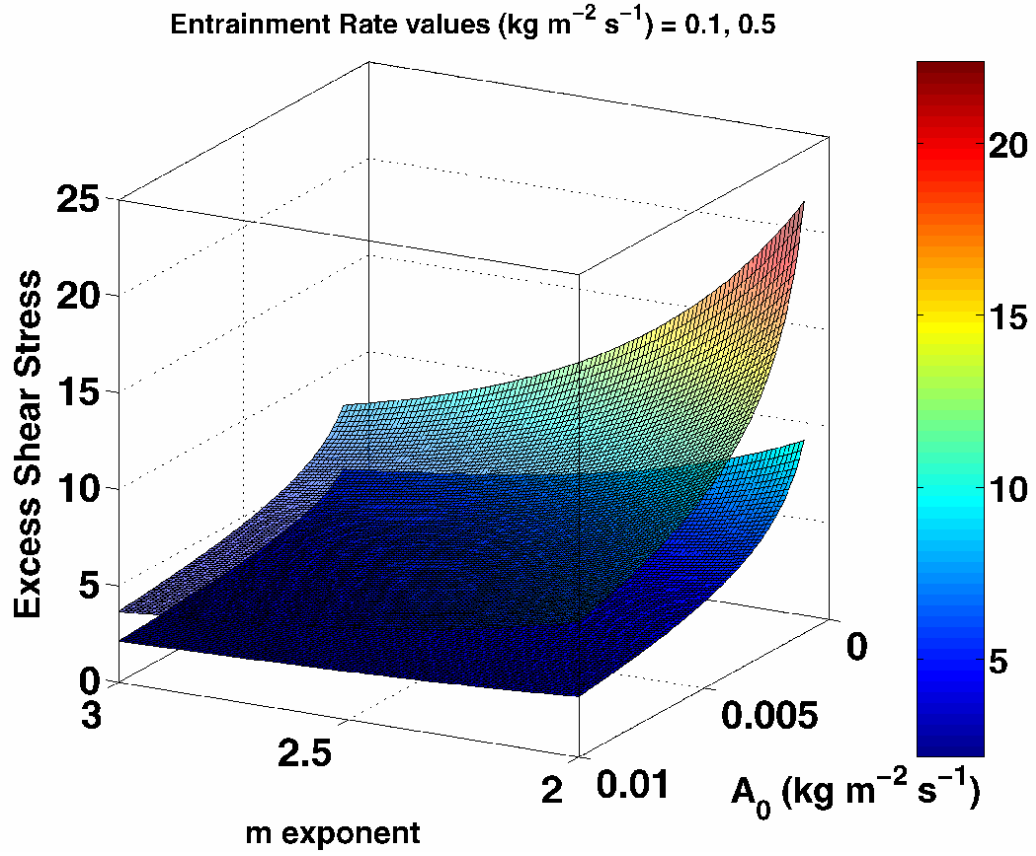


Figure 6. Isosurfaces of constant entrainment rate ( $E_B = 0.1, 0.5 \text{ kg m}^{-2} \text{s}^{-1}$ ) for a range of  $\tau^*$ ,  $A_0$ , and  $m$ . The colorbar represents the excess shear stress values.



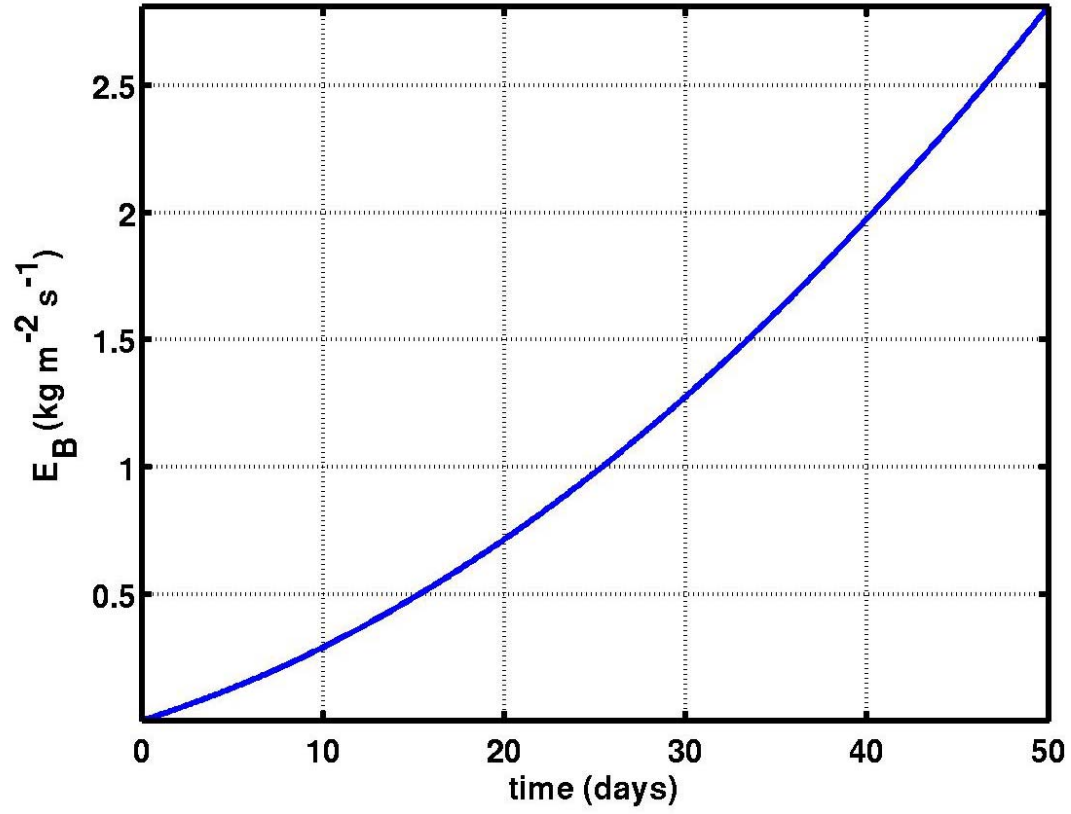


Figure 7.  $E_B$  ( $\text{kg m}^{-2} \text{s}^{-1}$ ) as a function of deposition time (days). The parameters  $A_\theta$  and  $\tau^*$  are set equal to 1.0. The  $m$  exponent is set to zero.

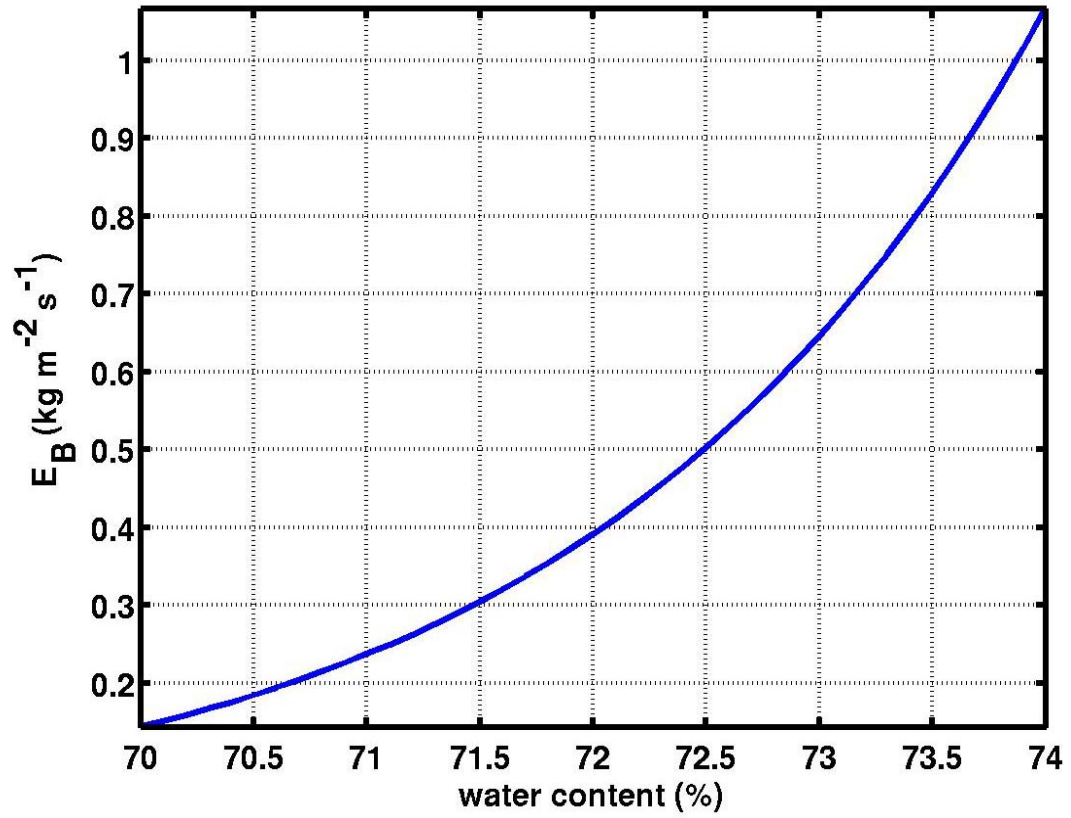


Figure 8.  $E_B$  ( $\text{kg m}^{-2} \text{s}^{-1}$ ) as a function of water content (%). The parameters  $A_0$  and  $\tau^*$  are set equal to 1.0. The  $m$  exponent is set to zero.

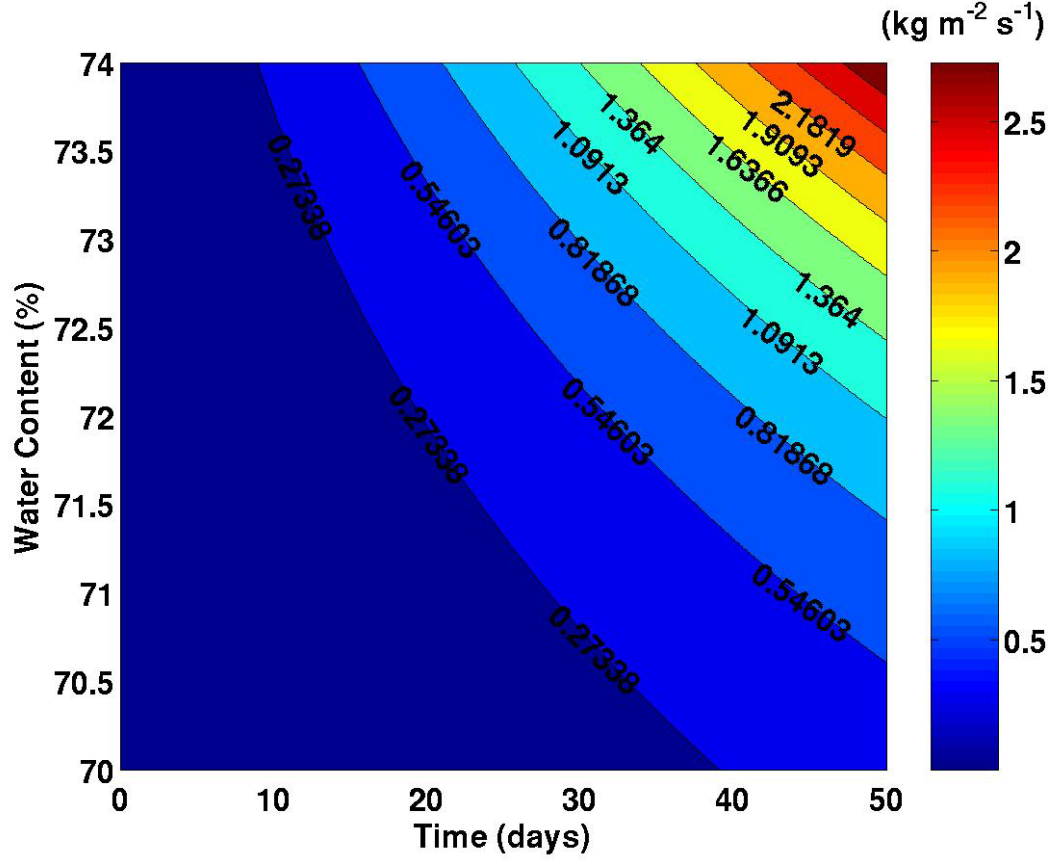


Figure 9. Contour plot of  $E_B$  ( $\text{kg m}^{-2} \text{s}^{-1}$ ) as a function of deposition time (in days) and water content (%). The parameters  $A_0$  and  $\tau^*$  are set equal to 1.0. The  $m$  exponent is set to zero.

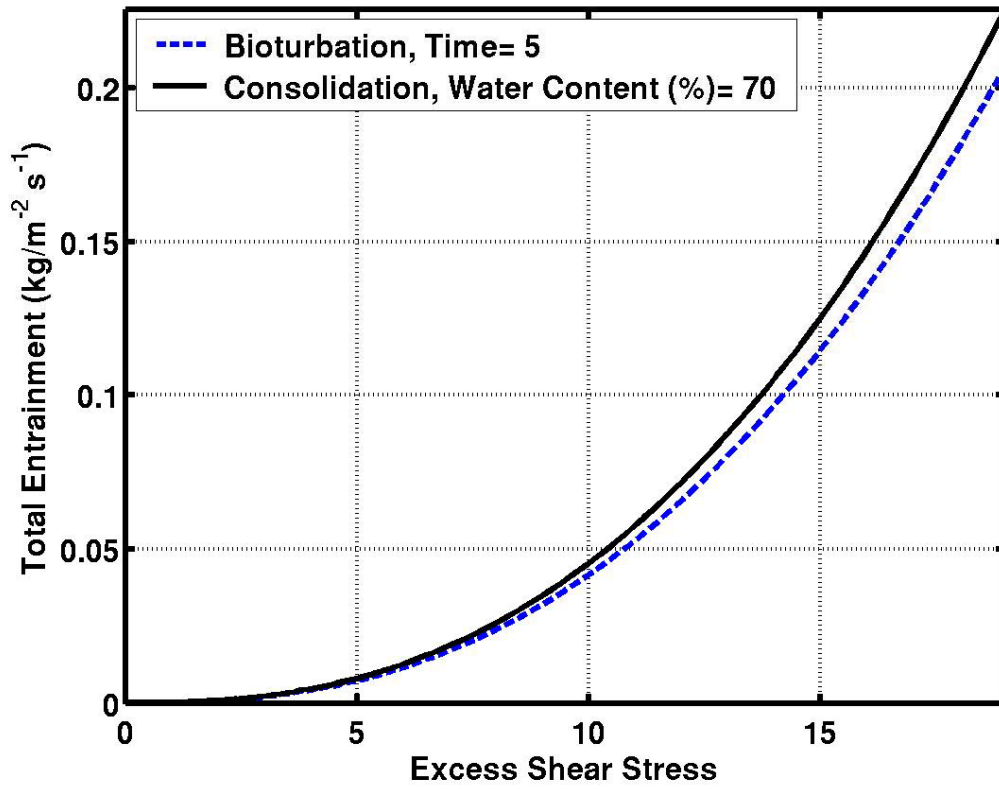


Figure 10. Entrainment rate ( $\text{kg m}^{-2} \text{s}^{-1}$ ) curves with bioturbation or consolidation activated.  $A_0 A_B$  is the factor for the dashed line and  $A_0 A_C$  is the factor for the solid line.  $A_0 = 0.001 \text{ (kg m}^{-2} \text{s}^{-1}\text{)}$  and the  $m = 2.5$ .

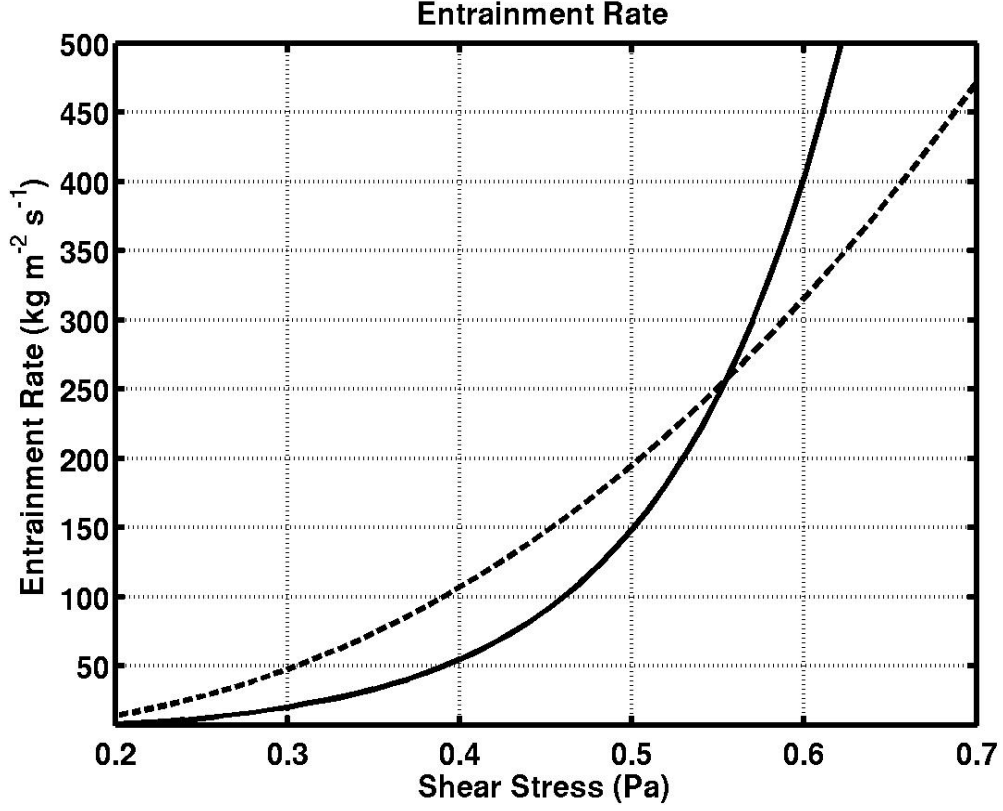


Figure 11. Power law and exponential entrainment rate ( $\text{kg m}^{-2} \text{s}^{-1}$ ) functions vs. shear stress. The constant parameters of the power law function (dashed line) are  $A_0 = 1.0 \text{ kg m}^{-2} \text{s}^{-1}$ ,  $\tau_c = 0.05 \text{ Pa}$ , and  $m = 2.4$ . The constant parameters of the exponential function (solid line) are  $\varepsilon_0 = 1.0 \text{ kg m}^{-2} \text{s}^{-1}$  and  $\alpha = 0.1 \text{ Pa}$ .

